Eur. Phys. J. B **37**, 363–368 (2004)
DOI: 10.1140/epjb/e2004-00067-3

THE EUROPEAN PHYSICAL JOURNAL B

Thermal conductivity and competing orders in d-wave superconductors

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Received 5 October 2003 / Received in final form 9 December 2003
Published online 2 April 2004 –

Abstract. We derive the expression for the thermal conductivity κ in the low-temperature limit $T \to 0$ in d-wave superconductors, taking into account the presence of competing orders such as spin-density wave, is-pairing, etc. The expression is used for analyzing recent experimental data in $La_{2-x}Sr_xCuO_4$. Our analysis strongly suggests that competing orders can be responsible for anomalies in behavior of thermal conductivity observed in those experiments.

PACS. 74.25.Fy Transport properties (electric and thermal conductivity, thermoelectric effects, etc.) – 74.72.Dn La-based cuprates – 74.72.-h Cuprate superconductors (high-Tc and insulating parent compounds)

The existence of four nodal points in d-wave superconductors provides rich and, sometimes, controllable dynamics of quasiparticle excitations at zero temperature. In particular, the expressions for electrical, thermal, and spin conductivity simplify considerably in the universal-limit $\omega \to 0$, $T \to 0$ [1,2]. It is noticeable that the role of the thermal conductivity κ is special: while vertex and/or Fermi-liquid corrections modify the bare, "universal", values of both electric and spin conductivities, the universal value of the thermal conductivity is not influenced by them [2]. It is:

$$\frac{\kappa_0}{T} = \frac{k_B^2}{3}\frac{v_F^2 + v_\Delta^2}{v_F v_\Delta}, \tag{1}$$

where v_F is a Fermi velocity, v_Δ is a gap velocity, and k_B is the Boltzmann constant (we use units with $\hbar = c = 1$). The basis for such a remarkably simple expression is that there is a finite density of states $N(0)$ of *gapless* quasiparticles down to zero energy [2,3]:

$$N(0) = \frac{2}{\pi^2 v_F v_\Delta}\Gamma_0 \ln\frac{p_0}{\Gamma_0}, \tag{2}$$

where $\Gamma_0 \equiv \Gamma(\omega \to 0)$, with $\Gamma(\omega)$ an impurity scattering rate, and $p_0 = \sqrt{\pi v_F v_\Delta}/a$ is an ultraviolet momentum cutoff (a is a lattice constant) [2]. Note that expression (1) itself is valid in the so-called "dirty" limit, $T \ll \Gamma_0$. Therefore, although this expression does not contain Γ_0 explicitly, a nonzero Γ_0 is crucial both for equations (1) and (2).

But what will happen if those quasiparticles become gapped? One may think that in that case both $N(0)$ and κ_0 are zero. However, as will be shown in this paper, they both are finite even in that case, if the impurity scattering rate is non-zero. In fact, it will be shown that they are:

$$\frac{\kappa_0^{(m)}}{T} = \frac{k_B^2}{3}\frac{v_F^2 + v_\Delta^2}{v_F v_\Delta}\frac{\Gamma_0^2}{\Gamma_0^2 + m^2} \tag{3}$$

and

$$N_m(0) = \frac{2}{\pi^2 v_F v_\Delta}\Gamma_0 \ln\frac{p_0}{\sqrt{\Gamma_0^2 + m^2}}, \tag{4}$$

where m a quasiparticle gap. The noticeable point is that, for all values of the gap up to $m \simeq \Gamma_0$, the suppression of both thermal conductivity and quasiparticle density is mild: $\kappa_0/\kappa_0^{(m)}$ and $N(0)/N_m(0)$ are of order one. However, the suppression in thermal conductivity rapidly becomes strong as m crosses this threshold. The second noticeable point is that, as we will discuss below, the gap m plays here a universal role and may represent different competing orders in d-wave superconductors, such as spin density wave, charge density wave, is-pairing, etc. Although their dynamics are different, expressions (3) and (4) for $\kappa_0^{(m)}$ and N_m are the same. This happens because, first, all those gaps m correspond to different types of "masses" in the Dirac equation describing nodal quasiparticle excitations, and, secondly, unlike electric and spin conductivities, the thermal conductivity $k_0^{(m)}$ is blind with respect to quantum numbers distinguishing those masses.

The expression $k_0^{(m)}$ corresponds to the dirty limit when $T \ll \Gamma_0$. In d-wave superconductors, Γ_0 can be as

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large as of order 1 K or even 10 K, and $k_0^{(m)}$ can be an important measurable characteristic there. Recently, two experimental groups have observed an anomalous behavior in the thermal conductivity in underdoped $La_{2-x}Sr_xCuO_4$ (Refs. [4,5] and Refs. [6,7]). One of the most interesting observations of experiment [6] is that at very low temperatures the value of the thermal conductivity in underdoped $La_{2-x}Sr_xCuO_4$ (LSCO) is *less* than the absolute minimum $k_{min}/T = 2k_B^2/3$ of expression (1) for k_0/T, corresponding to the isotropic case with $v_F = v_\Delta$. This puzzle can be naturally explained by utilizing the modified expression (3) with a nonzero m describing a competing order in the superconducting phase. We will discuss this and other results of experiments [4–7] below.

At subkelvin temperatures relevant to the low-T heat conduction experiments, we will use the continuum, low-energy, description for the nodal quasiparticles in the d-wave state. At each node, the quasiparticles are described by a two-component Nambu field. It will be convenient, following reference [8], to utilize four-component fields, by combining Nambu fields corresponding to the nodes within each of the two diagonal pairs. Thus we have two four-component Dirac fields. The corresponding representation for three Dirac matrices is

$$\gamma_0 = \sigma_1 \otimes I, \quad \gamma_1 = -i\sigma_2 \otimes \sigma_3, \quad \gamma_2 = i\sigma_2 \otimes \sigma_1, \tag{5}$$

where σ_i are the Pauli matrices and while the first factor in the tensor product acts in the subspace of the nodes in a diagonal pair, the second factor acts on indices inside a Nambu field. The matrices satisfy the algebra $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$, $g_{\mu\nu} = (1,-1,-1)$, $\mu,\nu = 0,1,2$.

We will consider quasiparticle gaps with the matrix structure $O_i = (I, i\gamma_5, \gamma_3, \gamma_3\gamma_5)$. Here the matrices γ_3 and γ_5, anticommuting with matrices γ_ν, are

$$\gamma_3 = i\sigma_2 \otimes \sigma_2, \quad \gamma_5 = \sigma_3 \otimes I. \tag{6}$$

Then, for each of the two four-component Dirac fields, the bare Matsubara Green's function can be written as

$$G_0(i\omega_n, \mathbf{k}) = \frac{1}{i\omega_n\gamma_0 - v_F k_1\gamma_1 - v_\Delta k_2\gamma_2 - m_i O_i}. \tag{7}$$

Therefore, different gaps m_i correspond to different types of Dirac masses. As was pointed out in references [8–11], these gaps represent different competing orders in low energy limit. In particular, the mass m_1, with $O_1 = I$, describes the (incommensurate) cos spin-density-wave (SDW), and the mass m_2, with $O_2 = i\gamma_5$, describes sin SDW. The masses m_3 and m_4, with $O_3 = \gamma_3$ and $O_4 = \gamma_3\gamma_5$, correspond to the id_{xy}-pairing and the is-pairing, respectively. One can also consider a gap corresponding to the charge-density-wave (CDW). In that case, one should introduce a Dirac mass term mixing the four-component Dirac fields corresponding to the two different diagonal pairs of the nodes. For simplicity, we will not consider it in this paper.

The scattering on impurities can be taken into account by introducing a Matsubara self-energy $\Sigma(i\omega_n)$, so that the dressed Green's function becomes $G(i\omega_n, \mathbf{k}) = G_0(i\omega_n - \Sigma(i\omega_n), \mathbf{k})$. As usual, retarded Green's function is obtained by analytically continuing Green's function G, $G^R(\omega, \mathbf{k}) = G(i\omega_n \to \omega + i\epsilon, \mathbf{k})$, and the impurity scattering rate is defined as $\Gamma(\omega) = -\mathrm{Im}\Sigma^{\mathrm{R}}(\omega)$. At low temperatures we take $\Gamma_0 \equiv \Gamma(\omega \to 0)$. The size of Γ_0 depends on the impurity density n_{imp} as well as on the scattering phase shift δ. Solving the Schwinger-Dyson equation for the self-energy in the self-consistent t-matrix approximation, one can find that in the unitary limit ($\delta = \pi/2$) the equation determining Γ_0 for a nonzero m_i has the form [12]

$$\Gamma_0^2 = \pi^2 v_F v_\Delta \tilde{\Gamma} \left[N_f \ln \frac{p_0^2}{\Gamma_0^2 + m_i^2} \right]^{-1}, \tag{8}$$

where N_f is the number of four-component Dirac fields and $\tilde{\Gamma} = n_{imp}/\pi\rho_0$ with ρ_0 the normal state density of states. Since $v_\Delta \sim \Delta_0$, the magnitude of the superconducting gap, the scattering rate Γ_0 is proportional to $\sqrt{\Delta_0\tilde{\Gamma}} \sim \sqrt{\Delta_0 n_{imp}}$.

The longitudinal dc thermal conductivity is calculated by means of the Kubo formula. In the bubble approximation, following the standard procedure, it can be expressed through the quasiparticle spectral function $A(\omega, \mathbf{k})$ as follows

$$\begin{aligned}\kappa^{(m)} = {} & \frac{\pi N_f}{8k_B T^2} \int_{-\infty}^{\infty} \frac{d\omega\omega^2}{\cosh^2 \frac{\omega}{2k_B T}} \int \frac{d^2k}{(2\pi)^2} \\ & \times \left\{ v_F^2 tr\left[\gamma_1 A(\omega,\mathbf{k})\gamma_1 A(\omega,\mathbf{k})\right] \right. \\ & \left. + v_\Delta^2 tr\left[\gamma_2 A(\omega,\mathbf{k})\gamma_2 A(\omega,\mathbf{k})\right] \right\}. \end{aligned} \tag{9}$$

Here the spectral function is given by the discontinuity of the fermion Green's function

$$A(\omega, \mathbf{k}) = -\frac{1}{2\pi i}\left[G^R(\omega + i\epsilon, \mathbf{k}) - G^A(\omega - i\epsilon, \mathbf{k}) \right]. \tag{10}$$

With Green's function at hand, we can calculate $A(\omega, \mathbf{k})$. For example, for the gap proportional to the unit Dirac matrix, it has the form ($m \equiv m_1$) [13]

$$\begin{aligned} A(\omega, \mathbf{k}) = {} & \frac{\Gamma_0}{2\pi E} \left[\frac{\gamma_0 E - v_F k_1\gamma_1 - v_\Delta k_2\gamma_2 + m}{(\omega - E)^2 + \Gamma_0^2} \right. \\ & \left. + \frac{\gamma_0 E + v_F k_1\gamma_1 + v_\Delta k_2\gamma_2 - m}{(\omega + E)^2 + \Gamma_0^2} \right], \end{aligned} \tag{11}$$

where $E(\mathbf{k}) = \sqrt{v_F^2 k_1^2 + v_\Delta^2 k_2^2 + m^2}$ is the quasiparticle energy. Substituting the last expression in equation (9) and taking the limit $T \to 0$, we arrive at

$$\begin{aligned} \frac{\kappa_0^{(m)}}{T} & = \frac{2\pi N_f k_B^2}{3} \int \frac{d^2k}{(2\pi)^2} \frac{\Gamma_0^2}{(E^2 + \Gamma_0^2)^2} \\ & = \frac{k_B^2 N_f}{6} \frac{v_F^2 + v_\Delta^2}{v_F v_\Delta} \frac{\Gamma_0^2}{\Gamma_0^2 + m^2}, \end{aligned} \tag{12}$$

i.e., we derived expression (3) for the thermal conductivity (in which $N_f = 2$). The result for three other gaps, m_2, m_3, and m_4, introduced above, is the same.

With the spectral function (11), the density of states (per spin)

$$N_m(\omega) = \frac{1}{2}\int \frac{d^2k}{(2\pi)^2} \mathrm{tr}\,[\gamma_0 A(\omega,\mathbf{k})] \tag{13}$$

is easily calculated

$$\begin{aligned} N_m(\omega) = \frac{N_f}{2\pi^2 v_F v_\Delta}\Bigg[&\Gamma_0 \ln \frac{p_0}{\sqrt{\Gamma_0^2 + (\omega - m)^2}} \\ &+ \Gamma_0 \ln \frac{p_0}{\sqrt{\Gamma_0^2 + (\omega + m)^2}} \\ &+ |\omega|\left(\frac{\pi}{2} + \tan^{-1}\frac{\omega^2 - m^2 - \Gamma_0^2}{2|\omega|\Gamma_0}\right)\Bigg]. \end{aligned} \tag{14}$$

It yields expression (4) for the density of states with zero energy. Therefore, in the presence of impurities, the quasiparticle band survives even for a finite m[1]. The physical reason for this is the formation of impurity bound states inside the gap [14]. Overlap between these states leads to impurity band supporting the quasiparticle heat (and electric) current.

The observation of a residual linear in T term in the thermal conductivity in cuprates ($YBa_2Cu_3O_{7-\delta}$ (YBCO) [15], $Bi_2Sr_2CaCu_2O_8$ (Bi-2212) [16] as well as LSCO [6]) is usually interpreted as a direct consequence of nodes in the gap. However, as it follows from equation (12), a subdominant order parameter, leading to a gap for nodal quasiparticles, does not exclude such a linear term in the thermal conductivity, although the latter does not have a universal form anymore[2].

Thus we conclude that nonperturbative dynamics, responsible for the creation of competing orders in the supercritical phase, can violate the universality in the thermal conductivity in the low temperature limit $T \to 0$. Recent experiments indicate that the existence of such competing orders is quite possible [17]. Several theoretical models have been proposed to describe this phenomenon (for a review, see Ref. [18]). As we will now discuss, using the expression for the thermal conductivity derived above, this phenomenon can be relevant for understanding recent experiments in $La_{2-x}Sr_xCuO_4$ [4–7].

The measurements of the thermal conductivity in LSCO at low temperature [4–7] showed the following characteristic features:

(a) at subkelvin temperatures, the value of κ/T decreases with x [4,6]. At temperature as low as 40 mK, the value of κ/T in some underdoped samples is either less than the absolute minimum $\kappa_{min}/T = 2k_B^2/3$ of expression (1) (for $x = 0.06$) or quite close to it (for $x = 0.07$ and $x = 0.09$) [6]. On the other hand, this anomalous behavior in the thermal conductivity disappears in overdoped samples ($x = 0.17$ and $x = 0.20$) [6];

(b) the evolution of κ/T across optimum doping is smooth [4,6];

(c) the thermal conductivity is sensitive to magnetic field. While in overdoped samples it increases with magnetic field, in underdoped samples the thermal conductivity decreases with increasing magnetic field [5,7]. The authors of references [5,7] describe this as a field-induced thermal metal-to-insulator transition;

(d) although remaining smooth, the evolution of κ/T across optimum doping becomes visibly faster with increasing magnetic field [5].

The results of item (a) can be easily understood if one assumes that there exists a competing order, described by the Dirac mass m, in the superconducting phase of underdoped LSCO. Then an appropriate value of m in expression (3) will provide the necessary suppression of the thermal conductivity. The fact that such an anomalous behavior in κ/T disappears with increasing x, in overdoped samples, can be understood if one assumes that the dynamical gap ("mass") m decreases with increasing x. As to this assumption, it is well-known in quantum field theory that, indeed, an increase of the fermion density often suppresses a dynamical Dirac mass. The reasons for that are simple. With increasing the fermion density, the screening effects become stronger and the quasiparticle interactions become weaker. In addition, at a sufficiently large quasiparticle density, the energy gain from creating a gap m in the quasiparticle spectrum will be surpassed by the energy loss of pushing up the energy of all states in the band above the gap. In the case of the model with Dirac fermions describing highly oriented pyrolytic graphite (HOPG) [19,20], this fact was explicitly shown in reference [20]. Although the present system is quite different from HOPG, that example supports plausibility of this assumption.

It is tempting to speculate that the dynamical gap m disappears close to optimum doping ($x_0 = 0.16$ in LSCO). A smooth evolution of κ/T across optimum doping then suggests that it could be a continuous phase transition with the scaling law of the form $m \sim (x_c - x)^\nu$ in the scaling region with $0 < (x_c - x)/x_c \ll 1$, where the critical value $x_c \simeq x_0$. The critical index $\nu = 1/2$ would correspond to the mean-field phase transition. In that case, there would be a kink in expression (3) at the critical point $x = x_c$. Indeed, since the thermal conductivity (3) depends on m^2, and there is a linear in m^2 term as $m^2 \to 0$, its derivative with respect to x will have a finite discontinuity at $x = x_c$ for $\nu = 1/2$. In the case of a non-mean-field continuum phase transition, with $\nu > 1/2$, the evolution of k/T across $x_c \simeq x_0$ would be smoother.

[1] Note that in the absence of impurities [$\Gamma_0 = 0$], we would get $N_m(\omega)|_{\Gamma_0=0} = (N_f/2\pi v_F v_\Delta)|\omega|\theta(\omega^2 - m^2)$, i.e., in that case, the mass m would lead to a gap in the density of states.

[2] Although the fact that opening of a gap for nodal quasiparticles leads to changes in $\kappa_0^{(m)}$ is natural from physical viewpoint, there has been a controversy concerning this point in the literature. For example, in the recent paper [12] the authors claim that in the limit $T \to 0$ the universal expression for the thermal conductivity, equation (1), survives even for gapped quasiparticles. Expression (12) derived above clearly shows that this is not the case.

This picture, with appropriate modifications, can survive in the presence of a magnetic field. In particular, the fact that in overdoped samples κ increases with magnetic field as $\sqrt{H}$ [5,7], implies that the dynamics in a magnetic field in overdoped samples is apparently conventional. Indeed, the $\sqrt{H}$ behavior is well described by semiclassical models [21]. This seems to suggest that there is no gap m (competing order) in overdoped samples.

The situation is different in underdoped samples. The magnetic field enhances the suppression in κ observed in the same samples at zero field (item (c) above). Moreover, the evolution κ/T across optimum doping becomes visibly faster with increasing H (item (d)). This suggests that magnetic field plays here the role of a catalyst, enhancing the gap m. For sufficiently large values of m, the suppression in κ will be so large that a sample effectively becomes a thermal insulator as was observed in experiments [5,7].

Microscopic dynamics responsible for creating competing orders can be quite sophisticated [18]. This issue is outside the scope of this paper. Here we will only comment on the role of a magnetic field as a catalyst in generating the gap m. In non-superconducting systems, it is well known that a magnetic field is indeed a strong catalyst in generating gaps (masses) for Dirac fermions [22]. In particular, this effect was studied in the model describing HOPG [19,20]. It is clear, however, that the dynamics in the vortex phase of d-wave superconductors is very different and the question about the relevance of a magnetic field for generating (or enhancing) a quasiparticles gap there is still open. For example, while the authors of papers [23–25] believe that such a role for a magnetic field in that phase is plausible, the analysis of the authors of reference [26] indicates that the magnetic field can actually supress id_{xy} and is gaps in a d-wave state.

In this paper, we will use a heuristic approach and demonstrate that the experimental data in references [5,7] can be qualitatively understood if one requires a gap that is generated below a critical doping and increases with a magnetic field. To make this point to be transparent, we are looking for an ansatz for the gap $m(H,x)$ which would be as simple as possible. We assume that (a) the phase transition at the critical doping $x = 0.16$ is the mean-field (or nearly mean-field) one, and (b) the gap increases as $\sqrt{H}$ with the magnetic field (such a scale covariant dependence of m on H was first considered in d-wave superconductors in Ref. [23]). This leads us to the ansatz:

$$m(H,x) = (1 - x/0.16)^{1/2}\ \theta(0.16 - x)(m_0 + bE_H), \quad (15)$$

where θ is the step function, $E_H = \hbar v_F/2R = (\hbar v_F/2)\sqrt{eH/\hbar c}$ is a characteristic energy scale in the presence of a magnetic field in the vortex state ($2R$ is the average distance between vortices), and m_0 and b are free parameters. Taking $v_F = 2.5 \times 10^7$ cm/s for LSCO cuprates [7,27], we find $E_H = 38\ \mathrm{K} \times \sqrt{H(T)}$ where the field $H(T)$ is taken in teslas. The constant b is of order 1 (for numerical calculations we take $b = 2.2$). As to the parameter m_0 that determines the gap for $H = 0$, it can be found from the ratio $\kappa/\kappa_0 = 2/3$ (i.e., $\kappa/T \simeq 12\ \mu\mathrm{W\,K^{-2}\,cm^{-1}}$) for $x = 0.06$, $H = 0$ and $T \to 0$, as reported in reference [6]. Then, taking $\kappa/\kappa_0 = \kappa_0^m/\kappa_0$, with κ_0^m from equation (3), we get $m_0 = a\Gamma_0$ where the constant $a \simeq 0.9$.

Let us now calculate the thermal conductivity by using ansatz (15) for $m(H,x)$. The impurity bandwidth for LSCO is estimated to be $\Gamma_0 \simeq 25$–30 K [7,27] which is two orders of magnitude larger than for very clean $YBa_2Cu_3O_{6.99}$ samples. While in clean $YBa_2Cu_3O_{6.99}$ the scattering of quasiparticles from vortices must be taken into account, one can neglect the dependence of the width Γ_0 on the magnetic field (at least for not very high fields) in the case of rather dirty LSCO. On the other hand, the presence of a circulating supercurrent around vortices in the vortex state can be taken into account in the semiclassical approach by making the Doppler shift in quasiparticle energies, $\omega \to \omega - \mathbf{v}_s(\mathbf{r})\mathbf{k}$, [28] ($\mathbf{v}_s(\mathbf{r})$ is the superfluid velocity at a position $\mathbf{r}$ which depends on the form of vortices distribution). In this case, the local thermal conductivity $\kappa(r)$ has to be averaged over the unit cell of the vortex lattice [29],

$$\kappa(H,T) = \frac{1}{A}\int d^2r\,\kappa(r) = \int d\epsilon\,\mathcal{P}(\epsilon)\kappa(\epsilon,T), \quad (16)$$

where

$$\mathcal{P}(\epsilon) = \frac{1}{A}\int d^2r\delta(\epsilon - \mathbf{v}_s(\mathbf{r})\mathbf{k}) \quad (17)$$

is the vortex distribution, and $A = \pi R^2$ is the area of the vortex unit cell. We use the Gaussian distribution function $\mathcal{P}(\epsilon) = (1/\sqrt{\pi}E_H)\exp[-\epsilon^2/E_H^2]$ which is believed to be the most suitable distribution in the presence of high disorder [30]. Thus we need to calculate

$$\begin{aligned}\kappa(H,T) = {}& \frac{\pi N_f}{8k_BT^2}\int_{-\infty}^{\infty}\frac{d\omega\omega^2}{\cosh^2\frac{\omega}{2k_BT}}\int_{-\infty}^{\infty}d\epsilon\mathcal{P}(\epsilon)\int\frac{d^2k}{(2\pi)^2}\\ &\times\left\{v_F^2 tr\left[\gamma_1 A(\omega-\epsilon,\mathbf{k})\gamma_1 A(\omega-\epsilon,\mathbf{k})\right]\right.\\ &\left.+\ v_\Delta^2 tr\left[\gamma_2 A(\omega-\epsilon,\mathbf{k})\gamma_2 A(\omega-\epsilon,\mathbf{k})\right]\right\} \quad (18)\end{aligned}$$

(compare with Eq. (9)).

Taking the limit $T \to 0$ in the last equation, we arrive at the following expression:

$$\begin{aligned}\frac{\kappa(H,0)}{\kappa_0} = {}& \frac{1}{2}\int_{-\infty}^{\infty}d\epsilon\mathcal{P}(\epsilon)\left[1 + \frac{\epsilon^2 - m^2 + \Gamma_0^2}{2|\epsilon|\Gamma_0}\right.\\ &\left.\times\left(\frac{\pi}{2} - \tan^{-1}\frac{\Gamma_0^2 + m^2 - \epsilon^2}{2|\epsilon|\Gamma_0}\right)\right], \quad (19)\end{aligned}$$

where we normalized the thermal conductivity on the universal value in equation (1).

In Figures 1 and 2 we present the ratio $\kappa(H,0)/\kappa_0$ calculated as a function of the magnetic field H (Fig. 1) and the doping x (Fig. 2). The form of these dependences is quite similar to that of experimental data presented in Figure 2 of reference [7] and in Figure 4 of reference [5], respectively.

At small values of the doping, $x = 0.06$ and $x = 0.13$ (low curves in Fig. 1), the thermal conductivity decreases with increasing field as a result of increasing

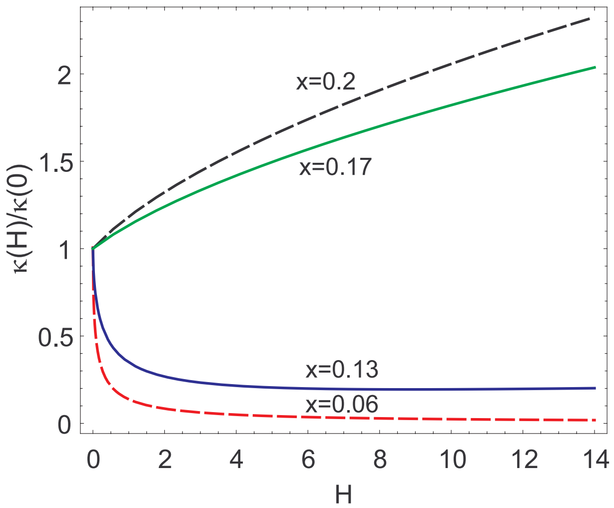

Fig. 1. $\kappa(H)/T$ [normalized to the value $\kappa(0)/T$] versus H at $T = 0$ and for the doping with the values $x = 0.06, 0.13, 0.17, 0.2$. The impurity width is $\Gamma_0 = 25$ K (for the upper curve ($x = 0.2$) $\Gamma_0 = 30$ K).

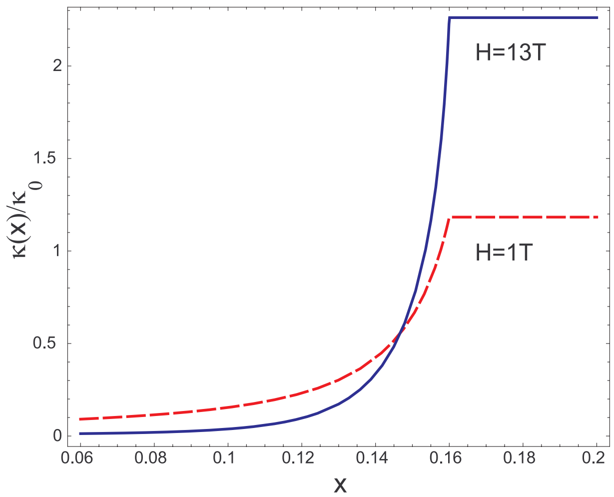

Fig. 2. Doping dependence of the $T = 0$ thermal conductivity κ [normalized to the universal value κ_0] for two values of the magnetic field $H = 1$ T (dotted curve) and $H = 13$ T (solid curve) and $\Gamma_0 = 25$ K.

the gap $m(H)$. For supercritical values of the doping ($x = 0.17; 0.2$ — upper curves in Fig. 1) the field dependence is approximately $\sqrt{H}$. This behavior is in accordance with the increase in quasiparticle population due to the Volovik effect that is valid even for gapped quasiparticles [31] when the vortex scattering is neglected.

Figure 2 shows the dependence of κ on the doping for two different values of the magnetic field. One can see the suppression of κ in the underdoped regime as a result of the presence of the magnetic-field-induced gap. Note that both curves grow fast near the critical doping $x_c = 0.16$ where the gap disappears. It is also noticeable that this growth is much faster for the $H = 13$ T curve than that for the $H = 1$ T curve. These facts agree with the experimental data [5] discussed in item (d) above.

Although the present analysis is based on the particular ansatz (15) for $m(H, x)$, one can expect that the main characteristics in the behavior of the thermal conductivity will retain qualitatively the same for a wide class of gaps $m(H, x)$ sharing the features that they are generated below a critical doping and increase with a magnetic field.

In conclusion, we derived the expression for the thermal conductivity in d-wave superconductors in the presence of competing orders. The derived expression (3) for $\kappa_0^{(m)}/T$ is simple and transparent. We also analyzed the dependence of the thermal conductivity on a magnetic field and a doping in the vortex state. Our results strongly suggest that the presence of competing orders can be crucial for understanding recent experiments in LSCO [4–7].

We thank E.V. Gorbar, D.V. Khveshchenko, V.M. Loktev, Yu.G. Pogorelov, S.G. Sharapov and I.A. Shovkovy for useful discussions. This work was supported by the Natural Sciences and Engineering Research Council of Canada. The research of V.P.G. was also supported in part by the SCOPES-projects 7 IP 062607 and 7 UKPJ062150.00/1 of Swiss NSF.

Note added in proof

Recently, in a different approach, a suppression of the thermal conductivity in the vortex phase of cuprates has been studied in M. Takigawa, M. Ichioka, K. Machida, `ArXiv: cond-mat/0306492`.

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